

拉氏变换练习题

一、用定义计算拉氏变换

$$(1) \ f(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$(2) \ f(t) = e^{-at}$$

$$(3) \ f(t) = \begin{cases} 0 & t < 0 \\ \sin \omega t & t \geq 0 \end{cases}$$

二、用性质计算拉氏变换

$$(4) \ \delta(t)$$

$$(5) \ \cos(\omega t)$$

$$(6) \ t$$

$$(7) \ \frac{t^2}{2}$$

$$(8) \ f(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 < t < a \\ 0 & t > a \end{cases}$$

$$(9) \ e^{-3t} \cdot \cos 5t$$

$$(10) \ e^{-2t} \cos\left(5t - \frac{\pi}{3}\right)$$

三、求初值

$$(11) \ \begin{cases} f(t) = t \\ F(s) = \frac{1}{s^2} \end{cases}$$

四、求终值

$$(12) \ F(s) = \frac{1}{s(s+a)(s+b)}$$

$$(13) \ F(s) = \frac{\omega}{s^2 + \omega^2}$$

五、求拉氏变换

$$(14) \ f(t) = 1 - e^{\frac{1}{r}t}$$

$$(15) \ f(t) = 0.03(1 - \cos 2t)$$

$$(16) \ f(t) = \sin\left(5t + \frac{\pi}{3}\right)$$

$$(17) \ f(t) = e^{-0.4t} \cos 12t$$

六、求初值和终值

$$(18) \ F(s) = \frac{3s^2 + 2s + 8}{s(s+2)(s^2 + 2s + 4)}$$

参考答案

一、用定义计算拉氏变换

$$(1) L[1(t)] = \int_0^\infty 1 \cdot e^{-st} dt = \frac{-1}{s} [e^{-st}]_0^\infty = \frac{-1}{s}(0 - 1) = \frac{1}{s}$$

$$(2) L[f(t)] = \int_0^\infty e^{-at} \cdot e^{-st} dt = \int_0^\infty e^{-(s+a)t} dt = \frac{-1}{s+a} [e^{-(s+a)t}]_0^\infty = \frac{-1}{s+a}(0 - 1) = \frac{1}{s+a}$$

(3)

$$\begin{aligned} L[f(t)] &= \int_0^\infty \sin \omega t \cdot e^{-st} dt = \int_0^\infty \frac{1}{2j} [e^{j\omega t} - e^{-j\omega t}] \cdot e^{-st} dt \\ &= \int_0^\infty \frac{1}{2j} [e^{-(s-j\omega)t} - e^{-(s+j\omega)t}] dt \\ &= \frac{1}{2j} \left[\frac{-1}{s-j\omega} e^{-(s-j\omega)t} \Big|_0^\infty - \frac{-1}{s+j\omega} e^{-(s+j\omega)t} \Big|_0^\infty \right] \\ &= \frac{1}{2j} \left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right] = \frac{1}{2j} \cdot \frac{2j\omega}{s^2 + \omega^2} = \frac{\omega}{s^2 + \omega^2} \end{aligned}$$

二、用性质计算拉氏变换

$$(4) L[\delta(t)] = L[1'(t)] = s \cdot \frac{1}{s} - 1(0^-) = 1 - 0 = 1$$

$$(5) L[\cos \omega t] = \frac{1}{\omega} L[\sin' \omega t] = \frac{1}{\omega} \cdot s \cdot \frac{\omega}{s^2 + \omega^2} = \frac{s}{s^2 + \omega^2}$$

$$(6) L[t] = L \left[\int 1(t) dt \right] = \frac{1}{s} \cdot \frac{1}{s} + \frac{1}{s} t \Big|_{t=0} = \frac{1}{s^2}$$

$$(7) L[t^2/2] = L \left[\int t dt \right] = \frac{1}{s} \cdot \frac{1}{s^2} + \frac{1}{s} \cdot \frac{t^2}{2} \Big|_{t=0} = \frac{1}{s^3}$$

$$(8) L[f(t)] = L[1(t) - 1(t-a)] = \frac{1}{s} - e^{-as} \cdot \frac{1}{s} = \frac{1 - e^{-as}}{s}$$

$$(9) L[e^{-3t} \cdot \cos 5t] = \frac{\hat{s}}{\hat{s}^2 + 5^2} \Big|_{\hat{s} \rightarrow s+3} = \frac{s+3}{(s+3)^2 + 5^2}$$

$$(10) L \left[e^{-2t} \cos \left(5t - \frac{\pi}{3} \right) \right] = L \left\{ e^{-2t} \cos \left[5 \left(t - \frac{\pi}{15} \right) \right] \right\} = \left\{ e^{-\frac{\pi}{15}\hat{s}} \frac{\hat{s}}{\hat{s}^2 + 5^2} \right\}_{\hat{s} \rightarrow s+2} = e^{-\frac{\pi}{15}(s+2)} \cdot \frac{s+2}{(s+2)^2 + 5^2}$$

三、求初值

$$(11) f(0) = \lim_{s \rightarrow \infty} s \cdot F(s) = \lim_{s \rightarrow \infty} s \cdot \frac{1}{s^2} = 0$$

四、求终值

$$(12) f(\infty) = \lim_{s \rightarrow 0} s \frac{1}{s(s+a)(s+b)} = \frac{1}{ab}$$

$$(13) f(\infty) = \sin \omega t \Big|_{t \rightarrow \infty} \neq \lim_{s \rightarrow 0} s \frac{\omega}{s^2 + \omega^2} = 0$$

五、求拉氏变换

六、求初值和终值